Mark Scheme (Results)

January 2022

Pearson Edexcel International A Level
in Statistics S2 (WST02) Paper 01

Question Number	Scheme	Marks
1 (a)	X = faults in a week $\Rightarrow X \sim \text{Po}(6)$	
	$[\text{P}(X \geqslant x) = 0.1528 \Rightarrow \text{P}(X \leqslant x-1)] = 0.8472$	M1
	Using tables $\text{P}(X \leqslant 8) = 0.8472 \Rightarrow x - 1 = 8$	M1
	$x = 9$	A1
		(3)
(b)	Y = faults in six weeks $\Rightarrow Y \sim \text{N}(36, 36)$	B1
	$\text{P}(Y < 32) = \text{P}\left(Z < \dfrac{31.5-36}{6}\right) \quad \left[= \text{P}(Z < -0.75)\right]$	M1 M1
	= 0.2266 awrt 0.227	A1
		(4)
(c)	W = Number of *poor weeks* $\Rightarrow W \sim \text{B}(50, 0.1528)$	B1
	$[\text{P}(W > 1)] = 1 - \text{P}(W \leqslant 1)$	M1
	$= 1 - \left(0.8472^{50} + 50 \times 0.1528 \times 0.8472^{49}\right)$	dM1
	= 0.99748… awrt 0.997	A1
		(4)

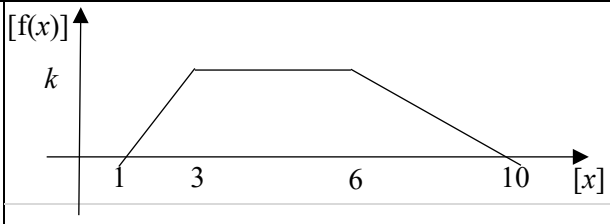
	Notes		**Total 11**
1 (a)	**M1**	Writing or using $1 - \text{P}(X \leqslant x - 1)$	
	M1	For 0.8472 May be implied by $x - 1 = 8$	
	A1	$x = 9$	
(b)	**B1**	Writing or using $\text{N}(36, 36)$ (May be implied by a correct standardisation expression)	
	M1	Standardising with 30.5/31/31.5/32/32.5/39.5/40/40.5/41/41.5, their mean and standard deviation (Allow ±)	
	M1	A fully correct standardisation. May be implied by ± 0.75	
	A1	awrt 0.227	
(c)	**B1**	Writing or using $\text{B}(50, 0.1528)$	
	M1	Writing or using $1 - \text{P}(W \leqslant 1)$ (Allow any letter)	
	dM1	Dependent on using binomial. Using $1 - \left[\text{P}(W = 0) + \text{P}(W = 1)\right]$ (implied by awrt 0.997 or 0.9975 or1 – awrt 0.00257) Using binomial may be implied by $(1-p)^{50} + {}^{n}C_{r} \times p \times (1-p)^{49}$ where p is a probability Condone ${}^{n}C_{r}$ missing	
	A1	awrt 0.997 or 0.9975	

Question Number	Scheme	Marks
2 (a)	$\mathrm{f}(x) = \begin{cases} \frac{1}{4k} & -k \leqslant x \leqslant 3k \\ 0 & \text{otherwise} \end{cases}$	M1 A1
		(2)
(b)	$[\mathrm{E}(X)] = k$	B1
		(1)
(c)	$[\mathrm{Var}(X)] = \frac{(3k - -k)^2}{12} = \frac{16k^2}{12}$ or $\left[\frac{x^3}{3}\ "\mathrm{f}(x)"\right]_{-k}^{3k} - ("k")^2$	M1
	$= \frac{4k^2}{3}$*	A1* cso
		(2)
(d)	$\mathrm{E}(X^2) = \mathrm{Var}(X) + \mathrm{E}(X)^2 = \frac{4k^2}{3} + ("k")^2$	M1
	$= \frac{7k^2}{3}$	A1
	$\mathrm{E}(3X^2) = 3\mathrm{E}(X^2) = 3 \times \frac{7k^2}{3} = 7k^2$	A1
		(3)

	Notes	**Total 8**
2 (a)	**M1**	For the 1st line of the pdf including the inequality, allow use of < instead of one/both $\leq$ signs
	A1	Fully correct, allow use of < instead of one/both $\leq$ signs. Allow equivalent for the 0 otherwise.
(b)	**B1**	Cao
(c)	**M1**	Use of $\mathrm{Var}(X) = \frac{(\beta - \alpha)^2}{12}$ or $\left[\frac{x^3}{3}\ "\mathrm{f}(x)"\right]_{-k}^{3k} - ("k")^2$
	A1* cso	Answer is given. Correct solution only with no incorrect working.
(d)	**M1**	Use of $\mathrm{E}(X^2) = \mathrm{Var}(X) + \mathrm{E}(X)^2$ ft their E(X) or $\left[\frac{x^3}{3}\ "\mathrm{f}(x)"\right]_{-k}^{3k}$ this integration may be seen in part (c) or part (d)
	A1	$\frac{7k^2}{3}$ (This must be seen in part (d)) May be implied by $7k^2$)
	A1	Cao

Question Number	Scheme	Marks
3 (a)	We can assume breakdowns are [rare], independent events occurring at a constant rate.	B1
		(1)
(b)	$H_0 : \lambda = 8$ $H_1 : \lambda \neq 8$	B1
		(1)
(c)	$X \sim \text{Po}(8)$	
	$P(X \leqslant 2) = 0.0138$ oe $P(X \leqslant 3) = 0.0424$ oe	M1
	$P(X \geqslant 14) = 0.0342$ oe $P(X \geqslant 15) = 0.0173$ oe	M1
	$X \leqslant 2 \cup X \geqslant 15$ oe	A1
		(3)
(d)	"0.0138" + "0.0173"	M1
	="0.0311"	A1ft
		(2)
(e)	"[4 is] not in the critical region"	M1
	So there is insufficient evidence that refurbishment has changed the mean breakdown rate	A1
		(2)

	Notes	Total 9
3 (a)	**B1**	A correct statement which include the words independent or constant rate or singly. No context needed
(b)	**B1**	Both hypotheses correct. Must be attached to H_0 and H_1 in terms of λ or μ.
(c)	**M1**	Use of $\text{Po}(8)$ to find the lower critical value. May be implied by either 0.0138 or 0.0424 or $X \leqslant 2$ if no probabilities shown (Calculator values: 0.01375… and 0.04238…)
	M1	Use of $\text{Po}(8)$ to find the upper critical value. May be implied by 0.0342 or 0.0173 or 0.9658 or 0.9827 or $X \geqslant 15$ if no probabilities shown (Calculator values: 0.03418… and 0.01725… and 0.96581… and 0.98274…)
	A1	$X \leqslant 2$ oe [$\cup$] $X \geqslant 15$ oe Condone the use of and/or Do not allow as probability statements Allow [0, 2] or [0, 3) and [15, ∞] or [15, ∞) or (14, ∞] or (14, ∞)
(d)	**M1**	Adding the two probabilities for their critical region
	A1ft	0.0311 Allow 3.11 or awrt 3.1[0] or awrt 0.031[0] ft their critical region
		NB 3.11 or 0.0311 or awrt 3.1[0] or awrt 0.031[0] will score 2/2
(e)	**M1**	A correct statement ft their critical region e.g. Do not reject H_0/Accept H_0/not significant – no context needed but do not allow contradicting non contextual comments
	A1	Correct conclusion in context. Must include rate/number of breakdown (Allow decreased for changed)
		NB Award M1 A1 for a correct contextual statement on its own

Question Number	Scheme	Marks
4 (a)		B1 B1
		(2)
(b)	$\frac{1}{2}(3+9)\times k=1$ or $\frac{1}{2}(3-1)k+(6-3)k+\frac{1}{2}(10-6)k=1$ or $\frac{1}{2}k\left[\frac{x^2}{2}-x\right]_1^3+k[x]_3^6+\frac{1}{4}k\left[10x-\frac{x^2}{2}\right]_6^{10}=1$	M1
	$k=\frac{1}{6}$ *	A1* cso
		(2)
(c)	$\int_1^x \frac{1}{12}(x-1)\,dx$ or $\int \frac{1}{12}(x-1)\,dx$ and using F(1) = 0	M1
	$\int_3^x \frac{1}{6}dx+"F(3)"$ or $\int \frac{1}{6}dx$ and using "F(3) = $\frac{1}{6}$"	M1
	$\int_6^x\left(\frac{5}{12}-\frac{1}{24}x\right)dx+"F(6)"$ or $\int\left(\frac{5}{12}-\frac{1}{24}x\right)dx$ and using either "F(6) = $\frac{2}{3}$" or F(10) = 1	M1
	$F(x)=\begin{cases}0 & x<1\\ \frac{1}{24}(x^2-2x+1) & 1\leqslant x\leqslant 3\\ \frac{1}{6}(x-2) & 3<x\leqslant 6\\ \frac{1}{48}(20x-x^2-52) \text{ or } 1-\frac{(10-x)^2}{48} & 6<x\leqslant 10\\ 1 & x>10\end{cases}$	A1oe A1oe A1 oe B1
		(7)
(d)	$P(X>E(X))=1-F\left(\frac{61}{12}\right)=1-0.51388...=0.4861...$ awrt 0.486	M1 A1 (2)
(e)	Since (d) < 0.5 [the mean is greater than the median] therefore positive (skew) or follow through their sketch in part (a)	M1 A1ft
		(2)

	Notes		Total 15
4(a)	**B1**	Correct shape. Must start and end on the x axis	
	B1	Fully correct including 1, 3, 6, 10 and k. Allow $^1/_6$ for k Ignore labels for x and f(x) and any extras e.g. $^k/_2$	
(b)	**M1**	Setting up the area of the trapezium = 1 or 2 triangles + a rectangle = 1 or a fully correct integration, including limits =1	
	A1* cso	Answer is given. Correct solution only with no incorrect working.	

(c)	**M1**	For a correct method to find the 2nd line Allow in terms of k

	M1	For a correct method to find the 3rd line, ft their F(3). If using + c method then ft their F(3) = $\frac{1}{6}$ Allow in terms of k
	M1	For a correct method to find the 4th line, ft their F(6). If using + c method then ft their F(6) = $\frac{2}{3}$ Allow in terms of k
	A1	2nd line correct including inequality. Allow < instead of $\leqslant$
	A1	3rd line correct including inequality. Allow < instead of $\leqslant$
	A1	4th line correct including inequality. Allow < instead of $\leqslant$
	B1	1st and 5th line correct. Allow "otherwise" for the range on the 1st or 5th line but not both. All 5 lines must be in terms of the same letter.
(d)	**M1**	For use of $1-\mathrm{F}\left(\frac{61}{12}\right)$ using the their line of F(x) for $3 < x \leqslant 6$. May use integration/area methods
	A1	awrt 0.486 Allow $^{35}/_{72}$
(e)	**M1**	For correctly comparing part (d) with 0.5 (may be implied by a correct comparison of mean and median (5)) do not allow mean is greater than the median on its own
	A1ft	For positive skew or ft their answer to part (d) Accept "no (or negligible) skew" following a reason that "mean $\approx$ median" Allow argument based on sketch in part (a)

Question Number	Scheme	Marks
5 (a)	B(n, 0.045)	B1
		(1)
(b)	Applicants are independent (no identical twins) or the proportion/probability identified as colour blind does not change over time	B1
		(1)
(c)	B(120, 0.045) $\Rightarrow$ Po(5.4)	B1
	$\mathrm{P}(X=5)=\dfrac{\mathrm{e}^{-5.4}\times 5.4^5}{5!}$	M1
	= 0.1728… awrt 0.173	A1
		(3)
(d)	Binomial with large n	B1
	and very small p	B1
		(2)
(e)	$\mathrm{H}_0: p=0.75 \quad \mathrm{H}_1: p\neq 0.75$	B1
	B(96, 0.75) $\Rightarrow$ N(72,18)	B1
	$Z=\dfrac{67.5-72}{\sqrt{18}}$ or $\dfrac{x\pm 0.5-72}{\sqrt{18}}$	M1
	= –1.06066… or $\dfrac{x+0.5-72}{\sqrt{18}}<-1.96$ or $\dfrac{x-0.5-72}{\sqrt{18}}>1.96$	A1
	P(z < –1.06) = 0.1444... / 0.1446 or CR < 63.2 awrt 0.144 or 0.145	A1
	There is insufficient evidence to reject H_0	dM1
	Insufficient evidence against Jaymini's claim	A1
		(7)
ALT	Let p be the probability of an applicant fail to become a pilot.	
	$\mathrm{H}_0: p=0.25 \quad \mathrm{H}_1: p\neq 0.25$	B1
	B(96, 0.25) $\Rightarrow$ N(24,18)	B1
	$Z=\dfrac{28.5-24}{\sqrt{18}}$ or $\dfrac{x\pm 0.5-24}{\sqrt{18}}$	M1
	= 1.06066… or $\dfrac{x+0.5-24}{\sqrt{18}}<-1.96$ or $\dfrac{x-0.5-24}{\sqrt{18}}>1.96$	A1
	P(z > 1.06) = 0.1444... / 0.1446 or CR > 32.8 awrt 0.144 or 0.145	A1
	There is insufficient evidence to reject H_0	dM1
	Insufficient evidence against Jaymini's claim	A1
		(7)

	Notes	**Total 14**
5 (a)	**B1**	For binomial with correct parameters n and 0.045
(b)	**B1**	For one of the given reasons. Must have context Allow equivalent statements Do not allow number for proportion/probability
(c)	**B1**	Using or writing Po(5.4)
	M1	For $\dfrac{\mathrm{e}^{-\lambda}\lambda^5}{5!}$ with any value for λ
	A1	awrt 0.173
		NB A correct answer with no incorrect working scores 3/3
(d)	**B1**	n is large (Allow number of trials for n)

	B1	p is small (Allow probability for p)
(e)	**B1**	Both hypotheses correct in terms of p or π Must be attached to H_0 and H_1
	B1	For writing or using N(72, 18) (May be implied by a correct standardisation expression)
	M1	Standardising using 67.5 or 67 or 66.5 or $x \pm 0.5$ with their mean and standard deviation (Allow ±)
	A1	awrt -1.06 (may be implied by awrt 0.144 or 0.145) or a correct standardisation with ±1.96 (ignore incorrect inequality symbol and allow =)
	A1	Using a probability route: awrt 0.144 or 0.145 or critical value of $z = \pm 1.96$ Using a critical region route: CR < 63.2
	dM1	Dependent on M1 A1. A correct statement – no context needed but do not allow contradicting non contextual comments. (Ignore any comparisons)
	A1	Correct conclusion in context. Must include the word claim. If they give an answer that refers to the claim then they must include the words applicants (oe), and pilots. No hypotheses then A0
		NB Award M1 A1 for a correct contextual statement on its own
ALT	**B1**	Both hypotheses correct in terms of p or π Must be attached to H_0 and H_1
	B1	For writing or using N(24, 18) (May be implied by a correct standardisation expression)
	M1	Standardising using 28.5 or 29 or 29.5 or $x \pm 0.5$ with their mean and standard deviation (Allow ±)
	A1	awrt 1.06 (may be applied by awrt 0.144 or 0.145) or a correct standardisation with ±1.96 (ignore incorrect inequality symbol and allow =)
	A1	Using a probability route: awrt 0.144 or 0.145 or critical value of $z = \pm 1.96$ Using a critical region route: CR < 32.8
	dM1	Dependent on M1 A1. A correct statement – no context needed but do not allow contradicting non contextual comments. (Ignore any comparisons)
	A1	Correct conclusion in context. Must include the word claim. If they give an answer that refers to the claim then they must include the words applicants (oe), and pilots. No hypotheses then A0
		NB Award M1 A1 for a correct contextual statement on its own

Question Number	Scheme	Marks
6 (a)	A sampling distribution is **all** the **values** of a **statistic** (obtained from a random sample) and the associated **probabilities** **or** the **probability distribution** of the **statistic (**under random sampling).	B1 (1)
(b)	$P(6)=\frac{6}{11}$ $P(7)=\frac{3}{11}$ $P(8)=\frac{2}{11}$	B1
	Totals (T) 12, 13, 14, 15, 16	B1
	(6, 6) (6, 7) (6, 8) (7, 6) (7, 7) (7, 8) (8, 6) (8, 7) (8, 8)	B1
	$[P(T=12)=]\ "\left(\frac{6}{11}\right)"^2=\left[\frac{36}{121}\right]$	
	$[P(T=13)=]2\times "\left(\frac{6}{11}\right)"\times "\left(\frac{3}{11}\right)"=\left[\frac{36}{121}\right]$	M1
	$[P(T=14)=]2\times "\left(\frac{6}{11}\right)"\times "\left(\frac{2}{11}\right)"+"\left(\frac{3}{11}\right)"^2=\left[\frac{33}{121}\right]$	M1
	$[P(T=15)=]2\times "\left(\frac{3}{11}\right)"\times "\left(\frac{2}{11}\right)"=\left[\frac{12}{121}\right]$	M1
	$[P(T=16)=]\ "\left(\frac{2}{11}\right)"^2=\left[\frac{4}{121}\right]$	
	T: 12, 13, 14, 15, 16 $P(T=t)$: $\frac{36}{121}$, $\frac{36}{121}$, $\frac{33}{121}$, $\frac{12}{121}$, $\frac{4}{121}$	A1 (7)
(c)	$E(T)="12"\times"\frac{36}{121}"+"13"\times"\frac{36}{121}"+"14"\times"\frac{33}{121}"+"15"\times"\frac{12}{121}"+"16"\times"\frac{4}{121}"$	M1
	$=\frac{1606}{121}=\frac{146}{11}=13.272...$ awrt 13.3	A1 (2)

	Notes	Total 10
6 (a)	**B1**	A correct explanation with the words in bold
(b)	**B1**	Correct probabilities – may be seen in an equation or implied by a correct probability for $T=14$
	B1	All 5 totals correct with no extras
	B1	All 6 basic combinations correct, either seen or used (may be implied by correct probailities) Allow S for 6, M for 7 and L for 8
	M1	Correct method for one probability ft their P(6), P(7) and P(8) If these are not stated then they must be correct
	M1	Correct method for three of the five probabilities ft their P(6), P(7) and P(8) If these are not stated then they must be correct
	M1	Correct method for all five probabilities ft their P(6), P(7) and P(8) If these are not stated then they must be correct or 5 probabilities that add up to 1
	A1	cao Need not be in a table but probabilities must be attached to the correct total
(c)	**M1**	Use of $\sum tP(T=t)$ two or more products ft their table
	A1	awrt 13.3 (Allow $\frac{146}{11}$ oe)

Question Number	Scheme		Marks
7 (a)	$P(L \geqslant 4.5) \Rightarrow P(A \geqslant 20.25)$		
	$P(A \geqslant 20.25) = (30 - 20.25) \times \frac{1}{20}$		M1
	$= 0.4875$		A1
			(2)
(b)	$\text{Var}(L) = E(L^2) - E(L)^2$		
	$[E(L^2) = E(A)] = 20$		B1
		$g(L) = \begin{cases} \frac{L}{10} & \sqrt{10} \leqslant L \leqslant \sqrt{30} \\ 0 & \text{otherwise} \end{cases}$	
	$E(L) = E(\sqrt{A}) = \frac{1}{20}\int_{10}^{30} \sqrt{a}\ \mathrm{d}A$	$E(L) = \frac{1}{10}\int_{\sqrt{10}}^{\sqrt{30}} L^2\ \mathrm{d}L$	M1
	$= \frac{1}{20}\left[\frac{2}{3}a^{\frac{3}{2}}\right]_{10}^{30}$	$\frac{1}{10}\left[\frac{l^3}{3}\right]_{\sqrt{10}}^{\sqrt{30}}$	A1
	$= 4.4231\ldots$		A1
	$\text{Var}(L) = \text{"20"} - (\text{"4.4231..."})^2$		M1
	$= 0.4358\ldots$	awrt 0.436	A1
			(6)

	Notes		**Total 8**
7 (a)	**M1**	$(30 - 20.25) \times \frac{1}{20}$	
	A1	cao (Allow 0.488 or $\frac{39}{80}$)	
(b)	**B1**	For 20	
	M1	Attempt to integrate $\frac{1}{20}\int_{10}^{30} \sqrt{a}\ \mathrm{d}A$ or $\frac{1}{10}\int_{\sqrt{10}}^{\sqrt{30}} L^2\ \mathrm{d}L$ Ignore limits and accept any letter	
	A1	Fully correct integration. Accept any letter. Must have limits	
	A1	4.42 or better	
	M1	Use of $\text{Var}(L) = E(L^2) - E(L)^2$ ft their $E(L^2)$ and $E(L)$ provided Var $(L) > 0$	
	A1	awrt 0.436	